Quadrupole magnetic field measurement with a rotating coil setup

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The quadrupole magnets situated between the undulators in the XFEL beam line need to be characterized with respect to magnetic axis position and stability and higher order field errors [1]. This can be done with a rotating coil setup [2, 3] and this status report describes such a setup and some initial measurements and tests. The report focuses on the measurement of the relative position of the magnetic axis and its stability at different currents as well as higher order field errors. The absolute position of the magnetic axis relative to markers on the magnet yokes, i.e. fiducialization, will be in a later report.

The magnetic field in a quadrupole can be written as,

$$\vec{B}(x,y,z) = \vec{B}_x + \vec{B}_y = g(y-y_0)\vec{e}_x + g(x-x_0)\vec{e}_y \tag{1}$$

where g is the constant magnet field gradient. The magnetic axis is where the magnetic field is zero, located at (x_0, y_0). A coil of length l and radius (or width) R rotating with angular frequency ω parallel to the magnetic axis experiences a change in magnetic flux. This corresponds to an induced voltage, V, given by,

$$V = -glR\omega\left\{r\sin(\theta_1+\theta) - R\sin(2\theta_2+2\theta)\right\} \tag{2}$$

where r and θ_1 represent the position of the magnetic axis in the frame of the rotation while θ_2 is the angle between the coordinate frame of the rotation and the coordinate frame of the magnetic field. This means that the position of the magnetic axis in the frame of the magnetic field is given by (see figure 1).

$$\begin{aligned} x &= r\cos(\theta_1-\theta_2) \\ y &= r\sin(\theta_1-\theta_2) \end{aligned} \tag{3}$$

The first term in equation (2) is the dipole component and is proportional to the distance between the axis of rotation and the magnetic axis (r). The second term comes from the quadrupole field and is independent of the position of the coil in the magnetic field. Higher order field errors are possible and appear at respective mode number. This means that the induced voltage can be written as a Fourier series [4]

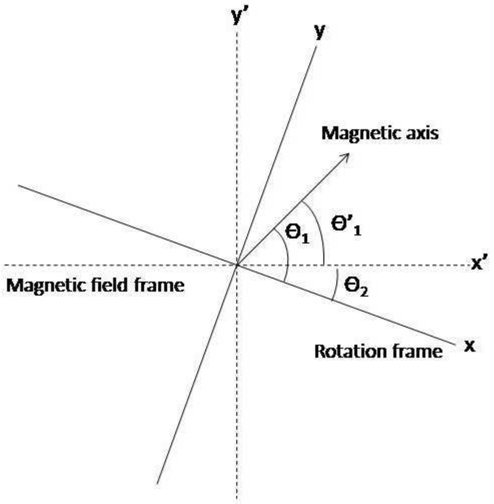

Figure 1: Rotation and magnetic field coordinate system

$$V = \sum_{n=1}^{\infty} p_n \sin(n\theta) + q_n \cos(n\theta) \tag{4}$$

with the Fourier coefficients given by,

$$\begin{matrix} p_n \\ q_n \end{matrix} = \frac{1}{\pi} \int_0^{2\pi} V(\theta) \begin{matrix} \sin n\theta \\ \cos n\theta \end{matrix} \, d\theta \tag{5}$$

From equation (2) and (4) follows that

$$\begin{aligned} r &= R\frac{\sqrt{p_1^2 + q_1^2}}{\sqrt{p_2^2 + q_2^2}} \\ \theta_1 &= \arctan(\frac{q_1}{p_1}) \\ \theta_2 &= \arctan(\frac{q_2}{p_2}) \end{aligned} \tag{6}$$

Thus, if the radius of the coil, R, is known, the position of the magnetic axis can be calculated directly from the Fourier coefficients.

Experimental setup

The rotating coil setup consists of a hollow steel shaft supported by ball-bearings. A 12 mm diameter epoxy G-10 rod extends 25 cm from the shaft. The rod has a long slit through it that holds two coils sitting side by side. The coils are 17 cm long, 6 mm wide and wound with 60 turns of 10 μm diameter copper wire. During measurement the coils are positioned symmetrically in the 12 cm long quadrupole magnet leaving 2.5 cm of coil on either side to accurately measure the magnetic field. The electrical signal from the coils passes through the shaft via a slip ring to a preamplifier before connection to a data acquisition card in a computer. An incremental encoder measures the position of the coils and returns 5000 pulses (TTL) per revolution (0.072° per step). Figure 2 shows the experimental setup ready for measurement with the coils inserted into the magnet. The position of the coils in the epoxy rod is shown in figure 3.

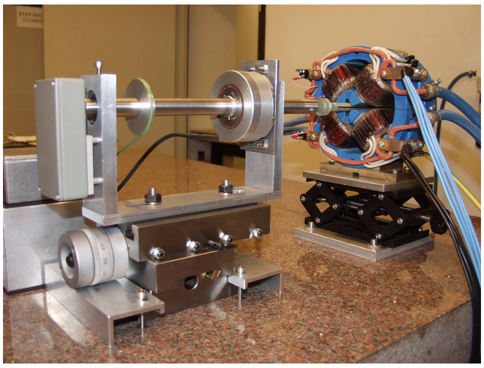

Figure 2: Rotating coil setup

The use of two coils fills two purposes. First, the average of the measurements of the magnetic axis position from the two coils cancels the effect of unwanted motion of the rod (wiggling) on position measurements. Second, the difference between the signals from the two coils partly cancels the dominating quadrupole term and allows for more sensitive measurement of weaker field error components. This is usually referred to as a compensated coil configuration.

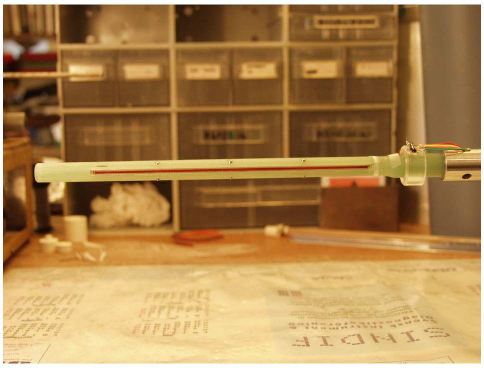

Figure 3: Coil in the G-10 rod

The coils rotate with a frequency of about 1 Hz, giving an encoder signal of 5 kHz. The signals (coil voltage and encoder TTL) are sampled at 12.5 kHz and voltage data are extracted at the encoder positions (the sample frequency limits the accuracy of the rotation angle to 1/5 of an encoder step, 0.0144°). The data acquisition card is not fast enough to record the data from one complete revolution before the next begins, therefore only every second revolution is recorded and the time between data points is about 2 seconds.

Results

The rotating coil setup has been tested on a quadrupole magnet borrowed from DESY. Figure 4 shows the magnetic axis position relative to the axis of rotation. Blue and red rings are data from the two coils respectively while the green rings are the average of the two signals. The data from individual coils are anti-correlated due to motion of the epoxy rod (and thereby the coils) with respect to the center of rotation. Movement of the shaft (and the center of rotation) will not influence the results in the same way. The Fourier coefficients calculated according to equation (5) give the average magnetic axis position during one revolution with respect to a moving shaft center.

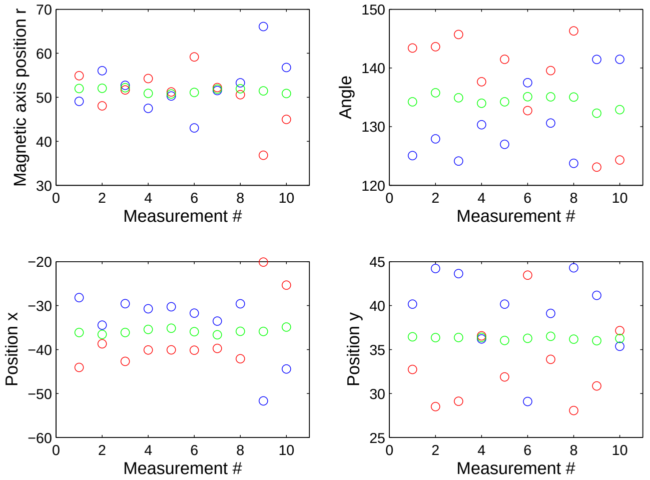

Figure 4: Magnetic axis position

Each data point in figure 4 represents one single revolution (of about 1 second) of the coils and the graph shows the evolution of the magnetic axis position during 20 seconds. Figure 5 shows the stability over a longer period (about 15 minutes). There is a clear, low-frequency (approximately 2 mHz) modulation of the signal. However, the statistical variation is still less than one micrometer (two standard deviations) which was the aim of the design of the rotating coil setup. Here, the magnetic axis position is $51.5 \pm 0.7\ \mu$m relative to the axis of rotation. The source of the modulation is not known but temperature variations will affect the measurement. To account for this, temperature sensors will be added to the setup and this will enable stability studies over longer periods of time.

The stability of the magnetic axis position at different currents is an important factor, and it is especially important when ramping the current up and down. Figure 6 shows such a measurement when the current in the magnet was increased from zero to 45 A in steps of 5 A (blue rings), then back to zero (red rings) and finally increased to 20 A (green rings) to complete the cycle.

Another aspect of magnet characterization is higher order field errors. Calculation of all Fourier coefficients from 1 to 50 based on data from the compensated

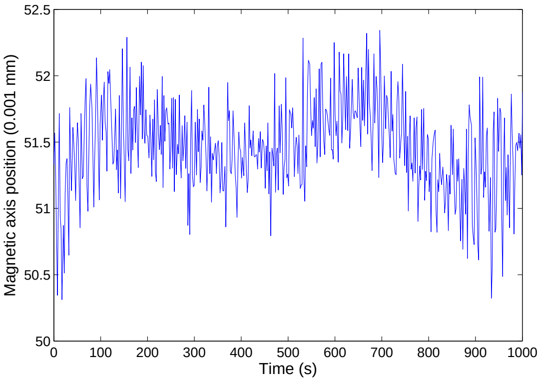

Figure 5: Magnetic axis stability over time

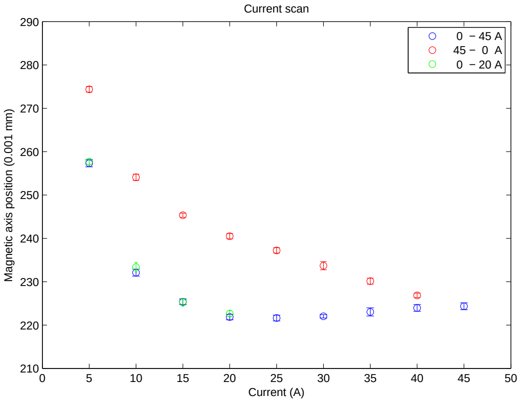

Figure 6: Magnetic axis position at different currents

coil is shown in figure 7. The quadrupole term (n = 2) still dominates the spectrum. The allowed error field components are n = 6, 10, 14... for a quadrupole magnet. It is not possible to distinguish n = 6 in the graph but n = 10 and n = 14 are clearly seen over the background level. Part of the background noise comes from resistance noise in the slip ring signal connector. The setup is currently being upgraded with a mercury wetted slip ring connector. This will hopefully reduce the background level and improve the data for error field components analysis.

The magnetic field coordinate system has its x-axis parallel to the table upon which the magnet rests. If the rotating coil setup moves parallel to the table a specific distance, the calculated position for the magnetic axis will move by the same amount. This can be used to calibrate the radius of the coils. In the analysis of the magnetic axis position, the radius R is equal to 6 mm, measured with a ruler. If this is the correct value then there will be a 1:1 relation between magnetic axis position calculated from recorded data and actual movement of the rotating coil setup. The result is shown in figure 8 and the least square fit gives a ratio of 0.97 which calibrates the coil radius to 6.2 mm.

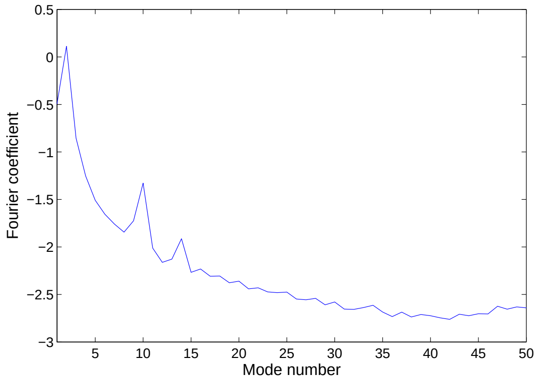

Figure 7: Fourier series components

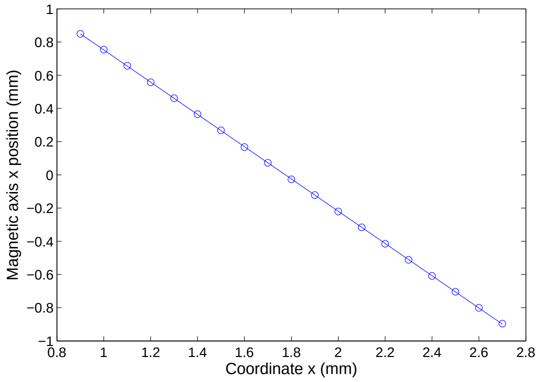

Figure 8: Calibration of the coil radius

During the 1.8 mm movement of the rotating coil in x-direction, the y-position only moved 11 μm, indicating that the magnetic field coordinate system is indeed parallel to the table. This calibration of the coil radius also makes it possible to determine the absolute position of the magnetic axis in the geometrical frame of the rotating coil setup and this is a first step to fiducialize the magnet.

Conclusions

The new rotating coil setup at MSL for magnetic field measurements allows precise measurement of the stability of the magnetic axis position. Measurements on a test quadrupole magnet show that over a period of 15 minutes the magnetic axis position remains within 0.7 μm, well within the requirements for the XFEL quadrupole magnets.

A current upgrade of the rotating coil setup with better slip ring signal connection, temperature sensors and stepper motor for precise motion control of the rotation will hopefully improve the magnetic field measurements and analysis even further.

Bibliography

[1] European XFEL Technical design report, edited by M. Altarelli et. al., DESY 2006.

[2] C. M. Spencer *et. al.*, SLAC-PUB-11473 (1998).

[3] C.S. Hwang *et. al.*, Nuclear Instruments and Methods in Physical Research A, **467-468**, 194 (2001).

[4] J. T. Tanabe, Iron dominated electromagnets, World Scientific publishing Co. Pte. Ltd (2005).